Congestion and Spatial Equilibrium Models

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Background

- NRC and Others have pointed to the need for ACE planning models to reflect:
  - The role of space in economic decision-making
  - The reality that decision-makers face multiple choices in shipping
    - Alternative modes
    - Alternative markets
    - Production decisions

Anderson & Wilson

- Five studies to date:
  - Congestion and Spatial Equilibrium in Barge Markets
  - Monopoly Railroads and Spatial Price Discrimination
  - ACE, S-TJ, and Full Spatial Equilibrium and Welfare Considerations
  - Market Power and Rivalry in Barge and Rail Markets:
    - Cournot
    - Betrand

Theoretical Background

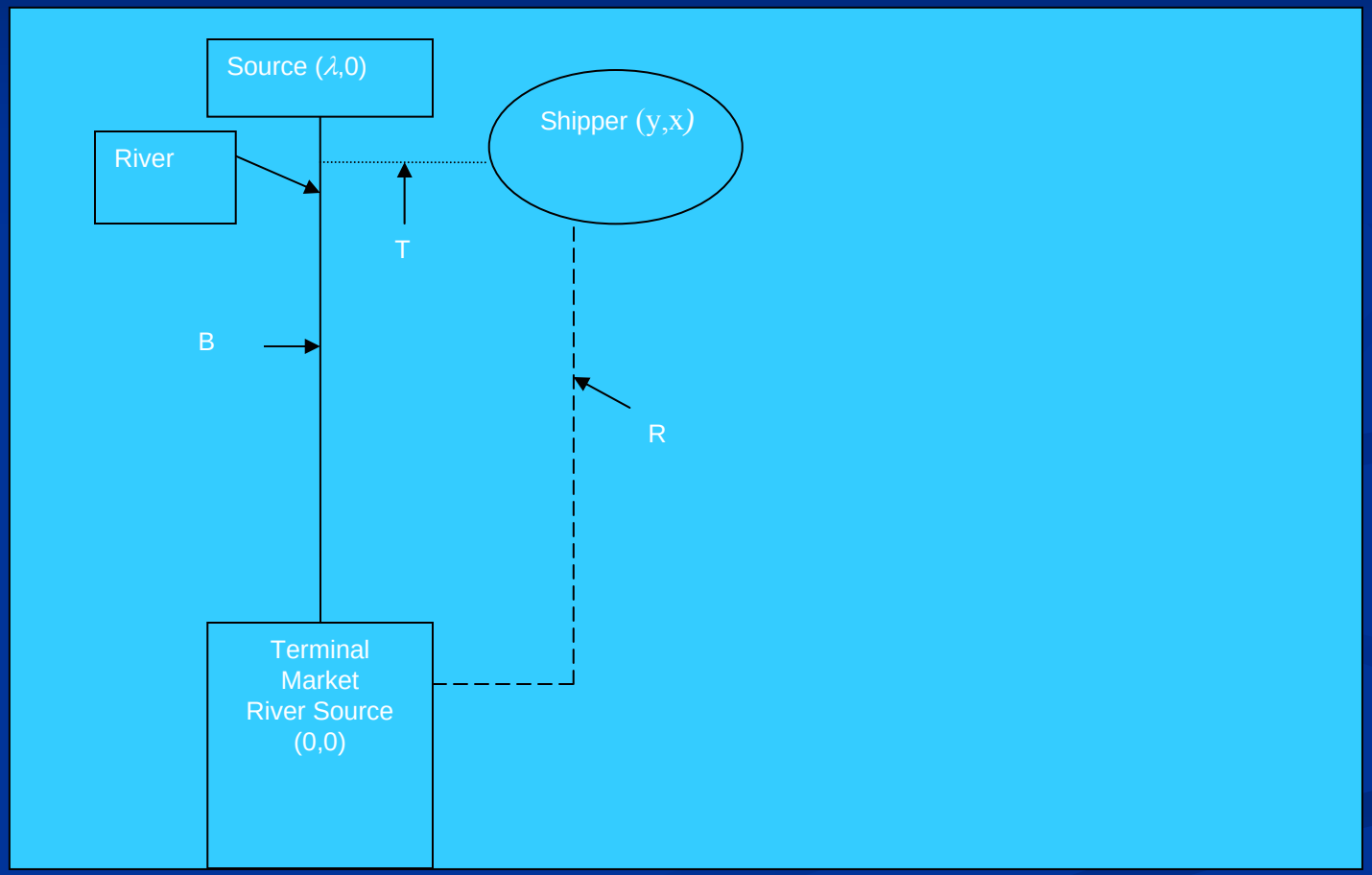
- Farmers geographically dispersed
- Truck-barge or rail
- Lock system and congestion
- Lock by-pass
- Endogenous price of transportation services

Basic model

- Terminal market at 0
- River runs NS along y-axis
- Transport metric is Manhattan
- River rate: b
- Truck rate: t
- Rail rate: r

$$b < r < t$$

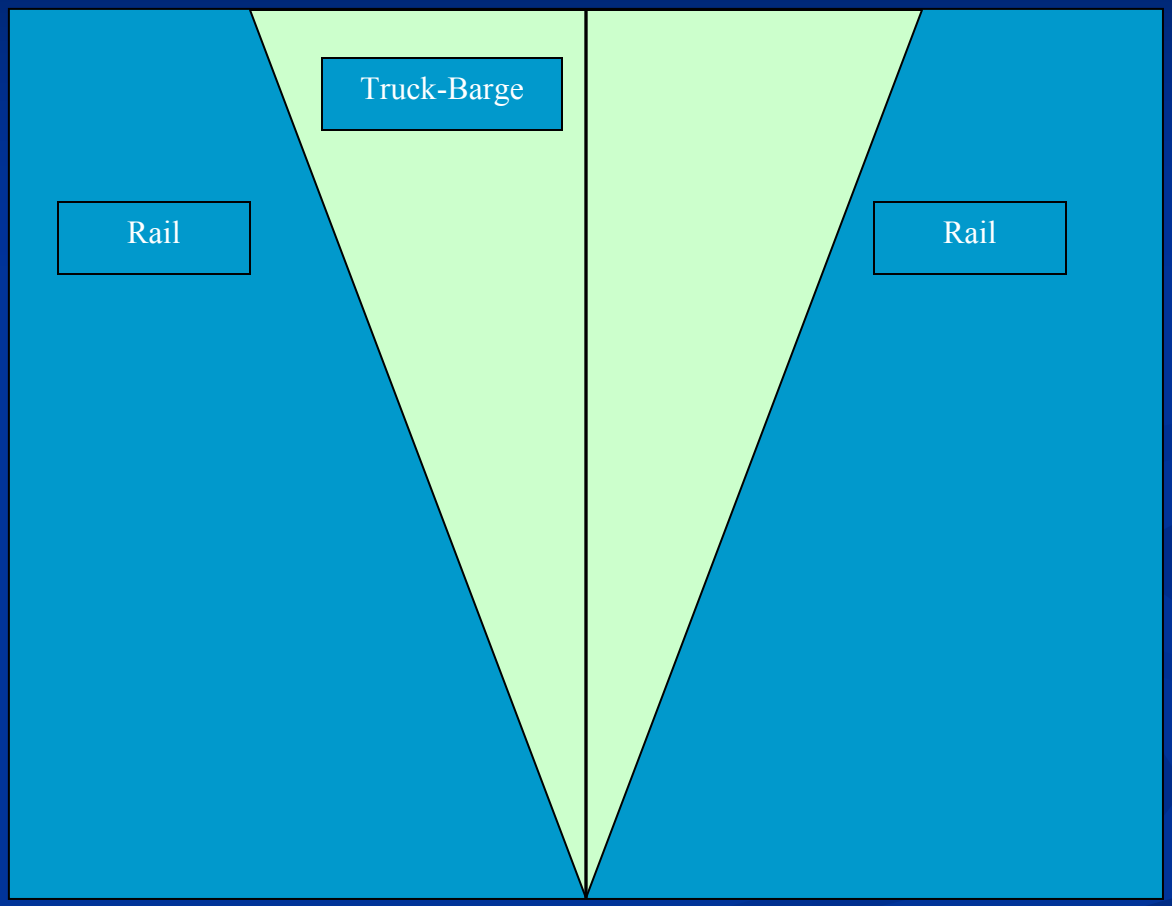
Figure 1-The Network



Truck-barge catchment area

- Truck-barge if: $rx + ry > tx + by$
- Indifference for: $y = x(t-r)/(r-b)$

Figure 2-Modal catchment areas



Fixed costs

- Now add fixed costs to shipment costs:

  For mode m:

  $$F_m + md, \quad m = b, r, t$$

  $$F_t < F_r < F_b$$

Figure 3-Fixed & Variable Shipment Costs

Rail Movements Dominated

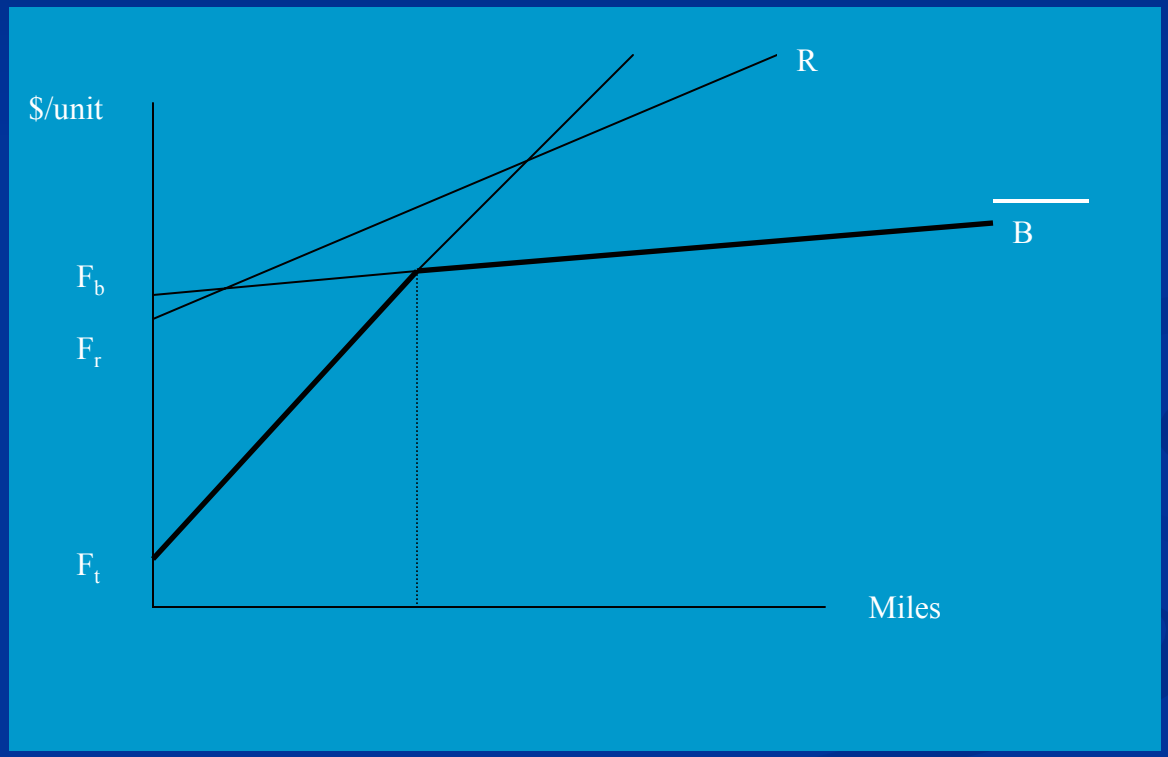


Figure 3-Fixed & Variable Shipment Costs

Rail Movements Not Dominated

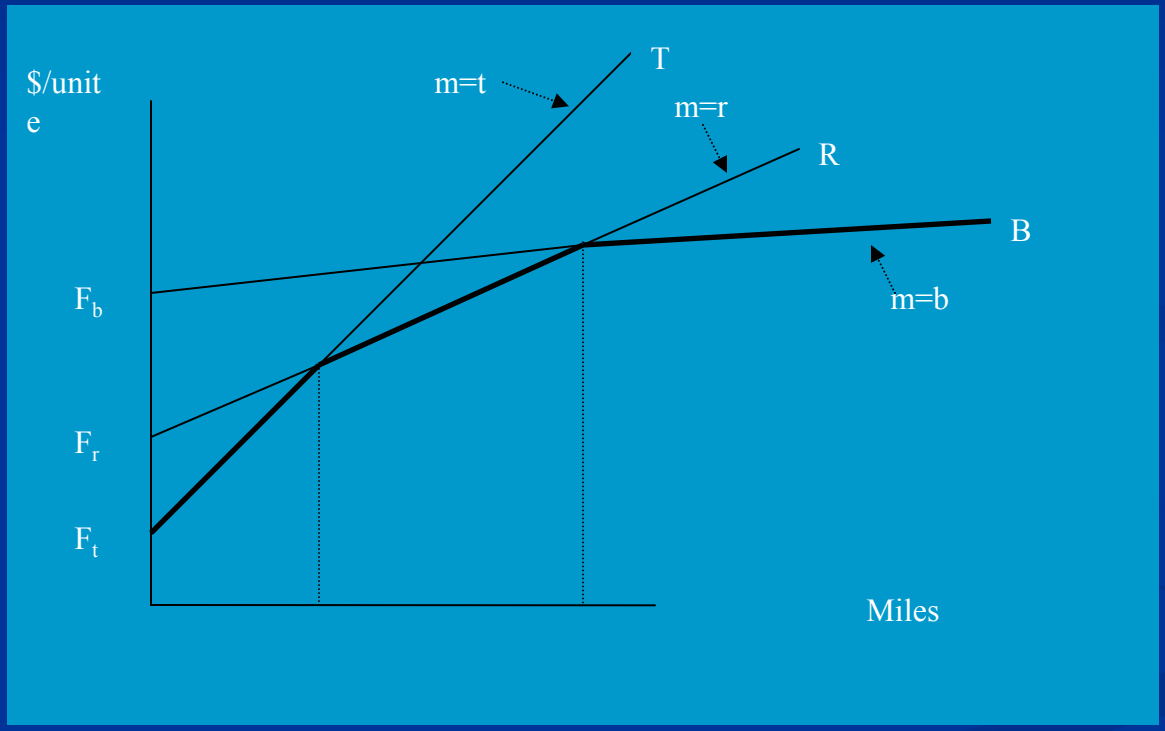
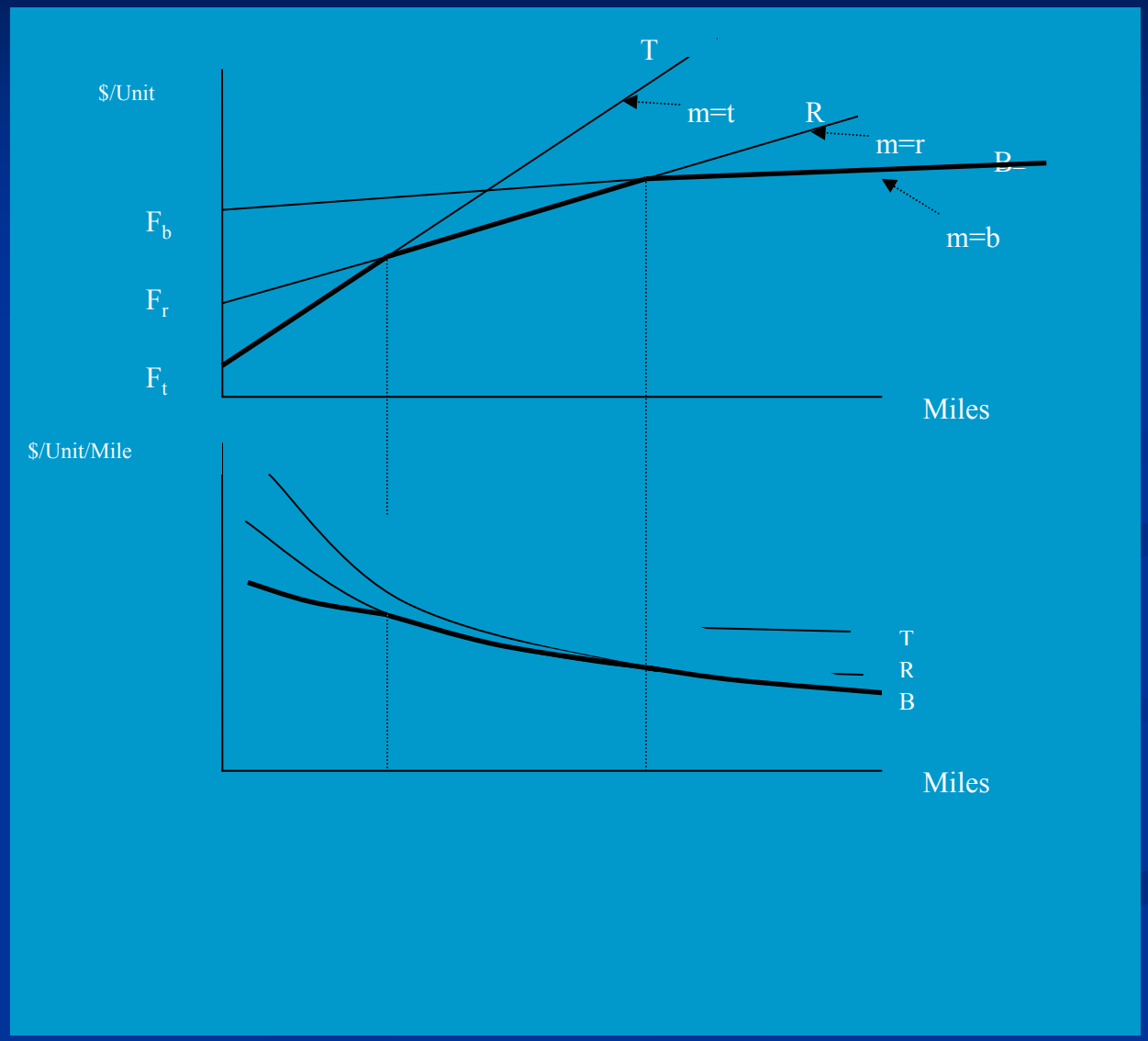
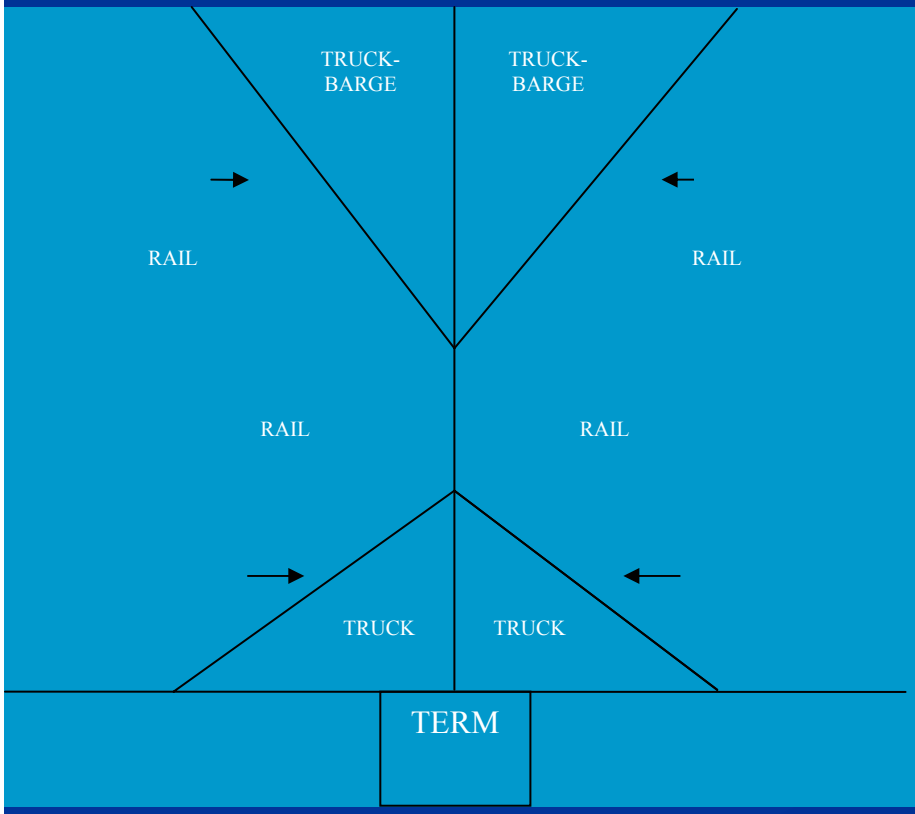


Figure 4-Rail Rate Tapers

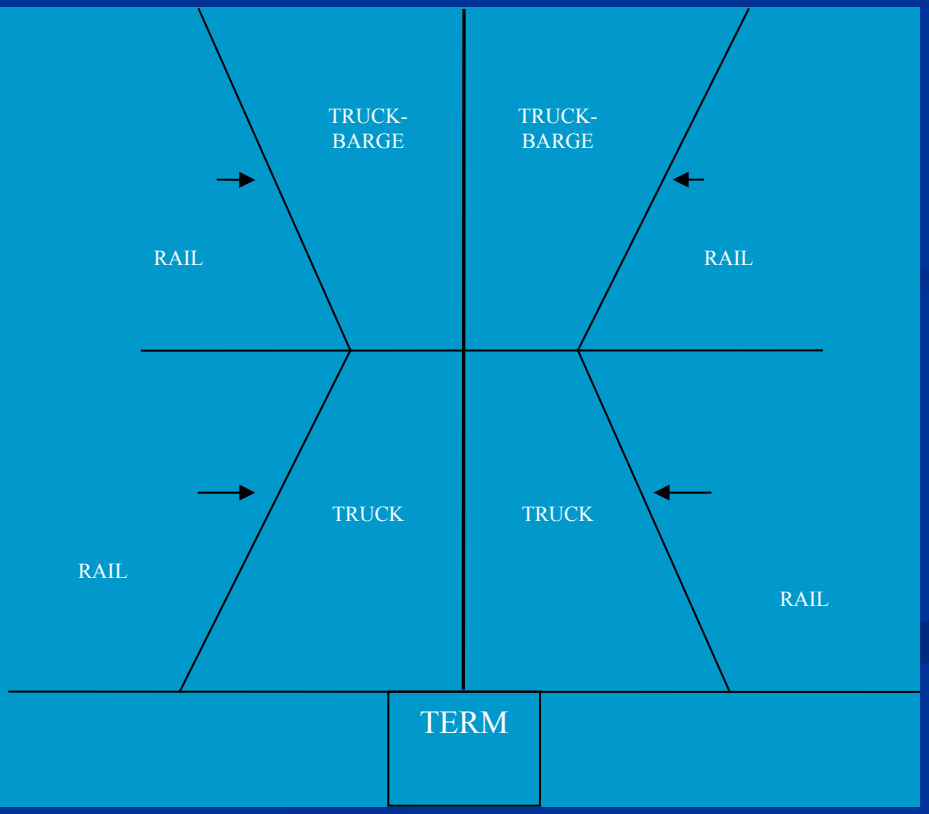


Figures 5 and 6
Catchment areas with fixed costs

Rail Not Dominated



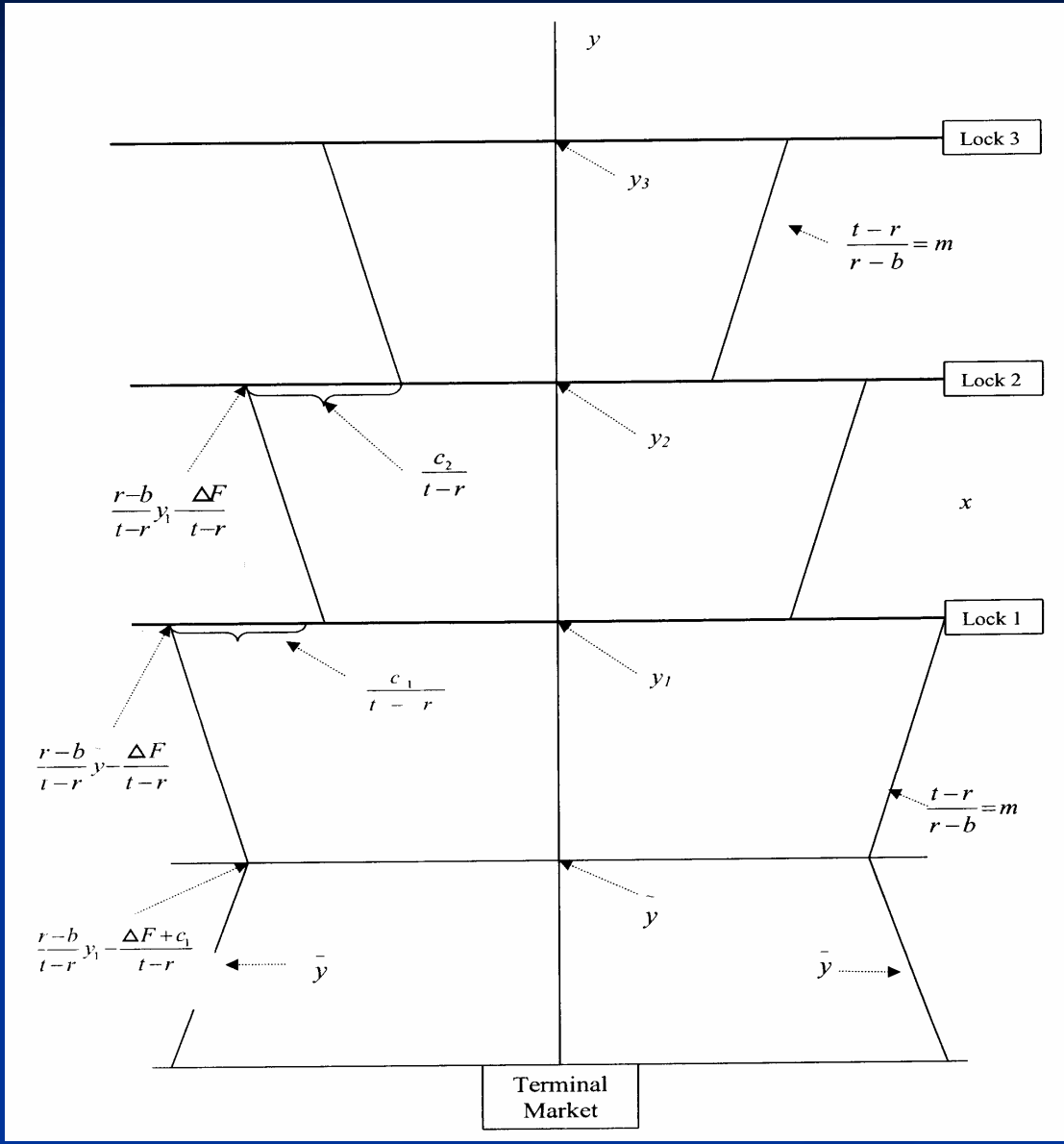
Rail Dominated



Locks

- Passing lock j costs C_j, $j = 1, \ldots, n$
- Truck-barge used from (y,x) if:
- $F_r + rx + ry > F_t + F_b + tx + by + \Sigma^j C_i$

Figure 7-Fixed Costs and Locks

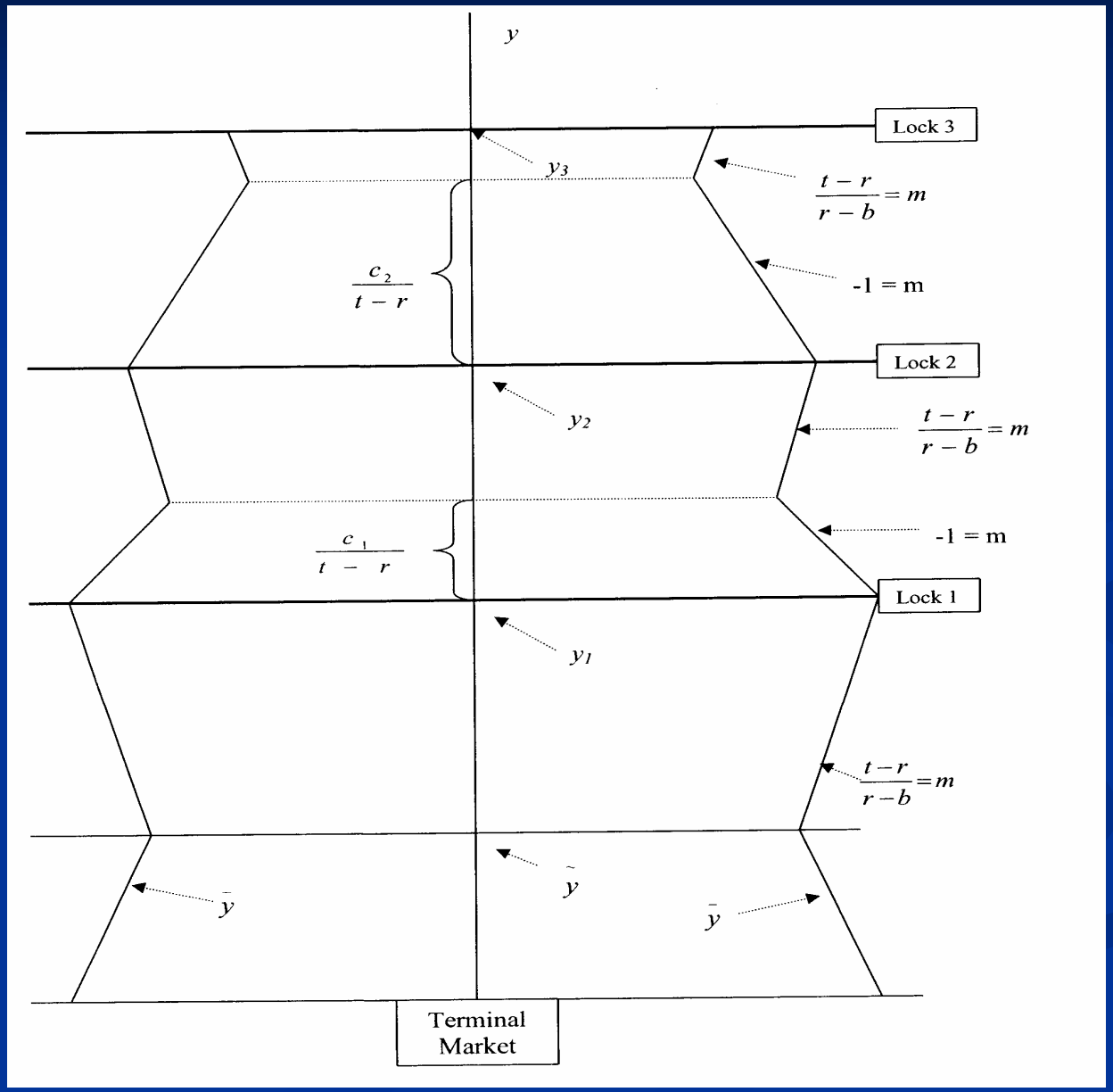


Lock by-pass

- Possible to by-pass one or several locks
- Use truck down to below the lock (enter at a river terminal)

Now advantage of rail *falls* closer to lock

Figure 8-Lock By-Pass



Congestion

- Depends on all shipping through lock, i.e., from all points up-river
- More traffic at locks lower down
- Single lock case: traffic depends on cost
- Cost depends on traffic. Equilibrium as FP
- Multi-lock case follows similar logic

Congestion with multiple locks

- Cost at Lock n depends on traffic emanating above it
- Traffic above Lock n depends on costs at all lower locks
- Cost at Lock n-1 depends on traffic from above n and between n-1 and n; traffic between n-1 and n depends on $C_1 \ldots C_{n-1}$
- Cost depends on traffic above; traffic depends on costs lower down

Demand and Costs

- $D_n = D_n (\Sigma^n C_i)$
- $D_j = D_j (\Sigma^j C_i)$
- $D_1 = D_1 (C_1)$
- $C_1 = C_1 (\Sigma^n D_i)$
- $C_j = C_j (\Sigma_{i=j..n} D_i)$
- $C_n = C_n (D_n)$
- Existence by fixed point argument

Existence

- Brouwer: cts mapping from a compact, convex set has a Fixed Point
- Assume D's and C's are cts and finite
- D's determine C's determine D's … i.e., maps "old" D's into new D's in a cts fashion.
- Hence a fixed point exists

Equilibrium uniqueness

- Suppose there were another.

Suppose $D_n' < D_n \Rightarrow C_n' < C_n$

- Then $C_j' > C_j$ for some j (to have $D_n' < D_n$)
- Then $D_{n-1}' < D_{n-1} \Rightarrow C_{n-1}' < C_{n-1}$

  etc. hence a contradiction.

- There exists a unique solution

Barge shipping rates

- Suppose # of barges is fixed – fixed time available
- Price will be determined by demand = S
- Rate p per unit time
- So $b = p\tau$, τ is time per mile

- $C_j = c_j + pT_j$ lock cost

c_j is money cost, T_j is time cost

Barge prices

- Existence of unique equilibrium p:

- Any given p determines unique set of D_i's as above
- As p falls, expect the D's to rise: but we need to account for the induced rise in congestion cost. Is it possible that they rise so much somewhere that overall barge demand falls?

Barge demand

- Suppose p fell and D_n fell.
- Then C_n would fall (2 reasons!)
- Then other C_i's would have to rise (D_n falls)
- Then D_{n-1} would fall, C_{n-1} would fall, etc.
- Thus D_n must rise
- A priori, C_n could fall or rise (rise if other costs fall enough)
- Work with volume of shipping through each lock, now show that it must rise for all locks

Shipping volumes

- We've shown D_n rises
- Suppose volume at n-1 fell (D_{n-1} falls a lot)
- C_{n-1} would fall.
- So all lower costs must rise to have D_{n-1} fall
- But then volume at n-2 would fall, etc.
- So all lower costs would fall, initial premise that D_{n-1} falls would be false, contradiction.
- Same argument applies for all subsequent locks
- Volume at each lock – and indeed demands – rise.

Unique barge price

- Just shown volumes rise through each lock
- Higher volumes use more barge time
- Total demand for barge time rises as p falls

- Hence a unique price
- Corresponding unique spatial demands and congestion levels though each lock

Comparative static properties

- Improve a lock (or locks)
- Reduce the congestion function at that lock
- Directly affects all shipments from further up: they increase
- This raises congestion at locks downstream
- This decreases shipments from points downstream

Conclusions

- Spatial Equilibrium with modal choice
- Series of congested points – existence and uniqueness of solution.
- Endogenous price of transport mode
- Model can be readily calibrated
- Basis for cost-benefit analysis

Future research directions

- Current work has uni-directional traffic

Future work will add in bi-directional barge movements

- Current work has but one terminal market

Future work will add in alternative terminal markets

- Current work has a distribution of shipment origins

Future work will integrate realistic distributions to represent locations of origins (the infrastructure will be added)

Future research directions

- Current work describes equilibrium for a single mode.
  - We are in the process of integrating equilibrium concepts for rail and truck. This requires:
    - A model of railroad pricing and capacity
    - A model of truck market equilibrium and capacity
- Current work describes congestion via a lock cost function.
  - We are in the process of integrating transit times for rail and truck too.
    - This allows congestion to be mode specific.
- Current work is purely theoretical.
  - We have the data and will be calibrating the model with transit, rate and cost functions.
  - This will allow simulations to be conducted on the model.

Railroad Pricing and Capacity

- **Purpose of Paper**
  - Describe competition between competitive truck-barge and monopoly railroad.
  - Model allows for:
    - Demanders located over space;
    - Multiple terminal markets (destinations);
    - Alternative pricing strategies by the railroad
    - Capacity constraints on the railroad

SETUP

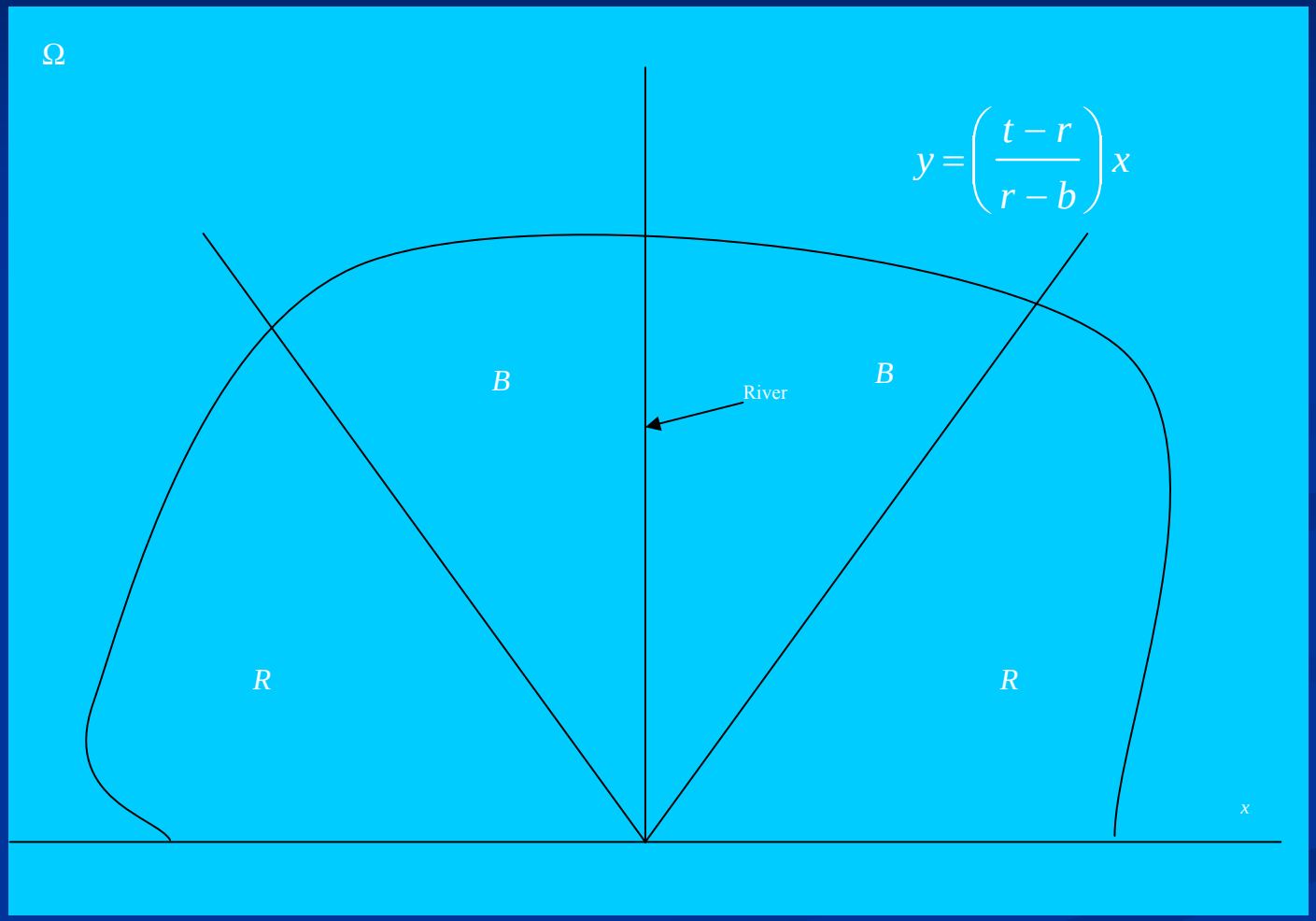
- Shippers are located over a space
- Shippers ship to a terminal point *(x=0, y=0)*.
- There is a river that runs N to S that can be accessed by truck and rail is available everywhere.
- Shipment sizes are unit.
- Reservation prices are high enough to insure that shipments are made.
- Truck, rail, and barge costs per unit distance are t > r > b.
- If rates reflect costs, the truck-barge market area is given by:

$$\mathbf{B} = \{(y,x) \in \Omega \ : \ t|x| + by \leq r|x| + ry\}$$

- The rail market area is given by:

$$\mathbf{R} = \{(y,x) \in \Omega \ : \ t|x| + by > r|x| + ry\}$$

Competitive Market Areas



Railroad's Profit Maximization

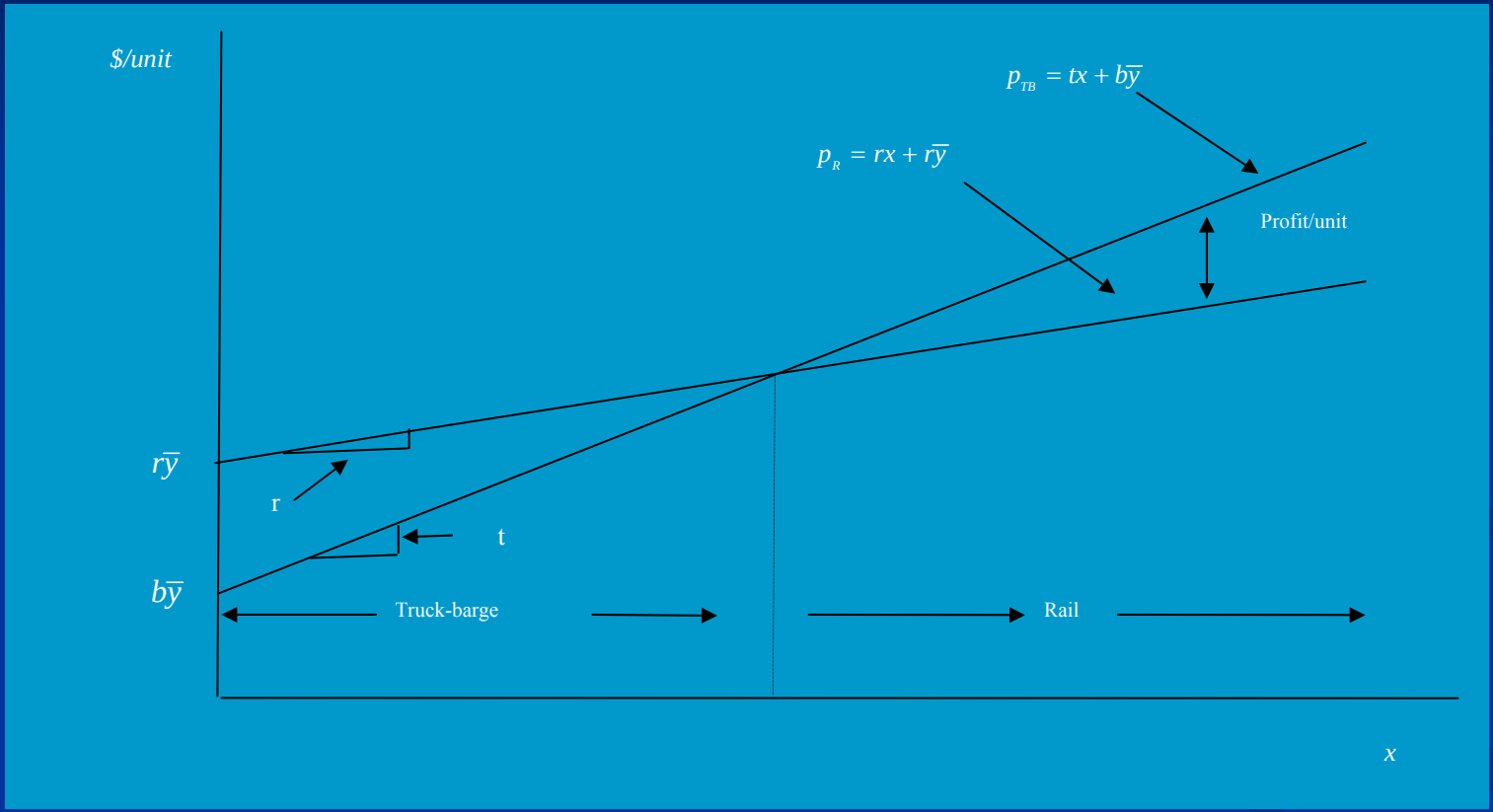
- ***Profit Maximize*** over its natural market ***R*** and does not want to serve any of the barge market ***B***.

- ***Independence Property***: Since $t>r$, the rail can choose its price at each point without regard to other points.

- ***Pricing:***
  - Railroad will not price below marginal cost and so will not serve any of ***B*** (the truck-barge) market.
  - Railroad will price all of ***R*** to "beat the competition". They price all points in ***R*** at the truck-barge cost.

- ***Results:***
  - 1. RR's profit is greatest furthest from the river;
  - 2. RR's market (gathering) area is unchanged from competitive model;
  - 3. The equilibrium is efficient, and market power only has distributional consequences (shippers to rail).
  - 4. Improvements to barge, reduces the rail rate and transfers rent from rail to farmers.

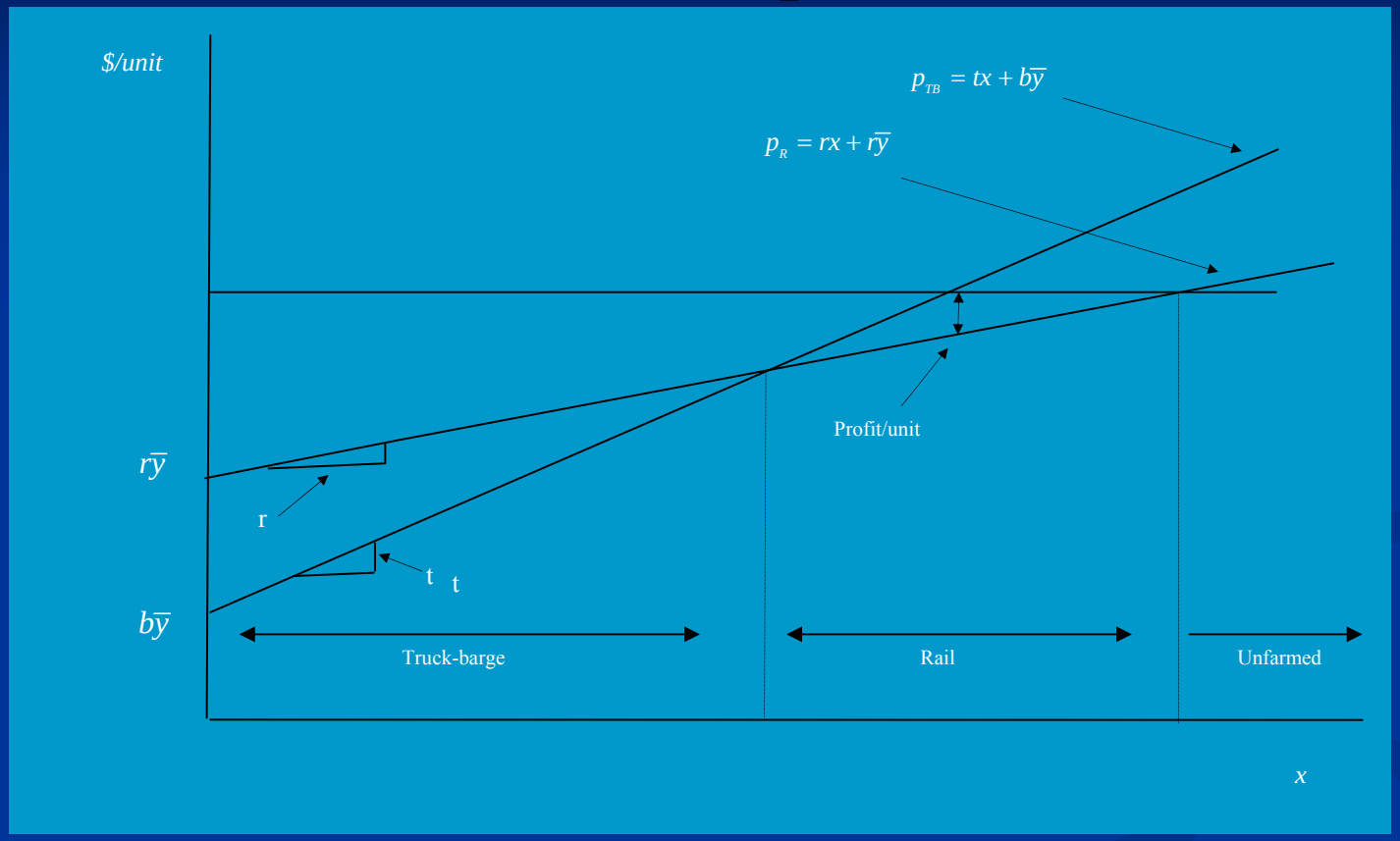
Rail Pricing and Distance from River



Results:

1. Rail dominated close to river and does not serve
2. Rail has market power when costs are lower (away from the river), and markups increase with distance.

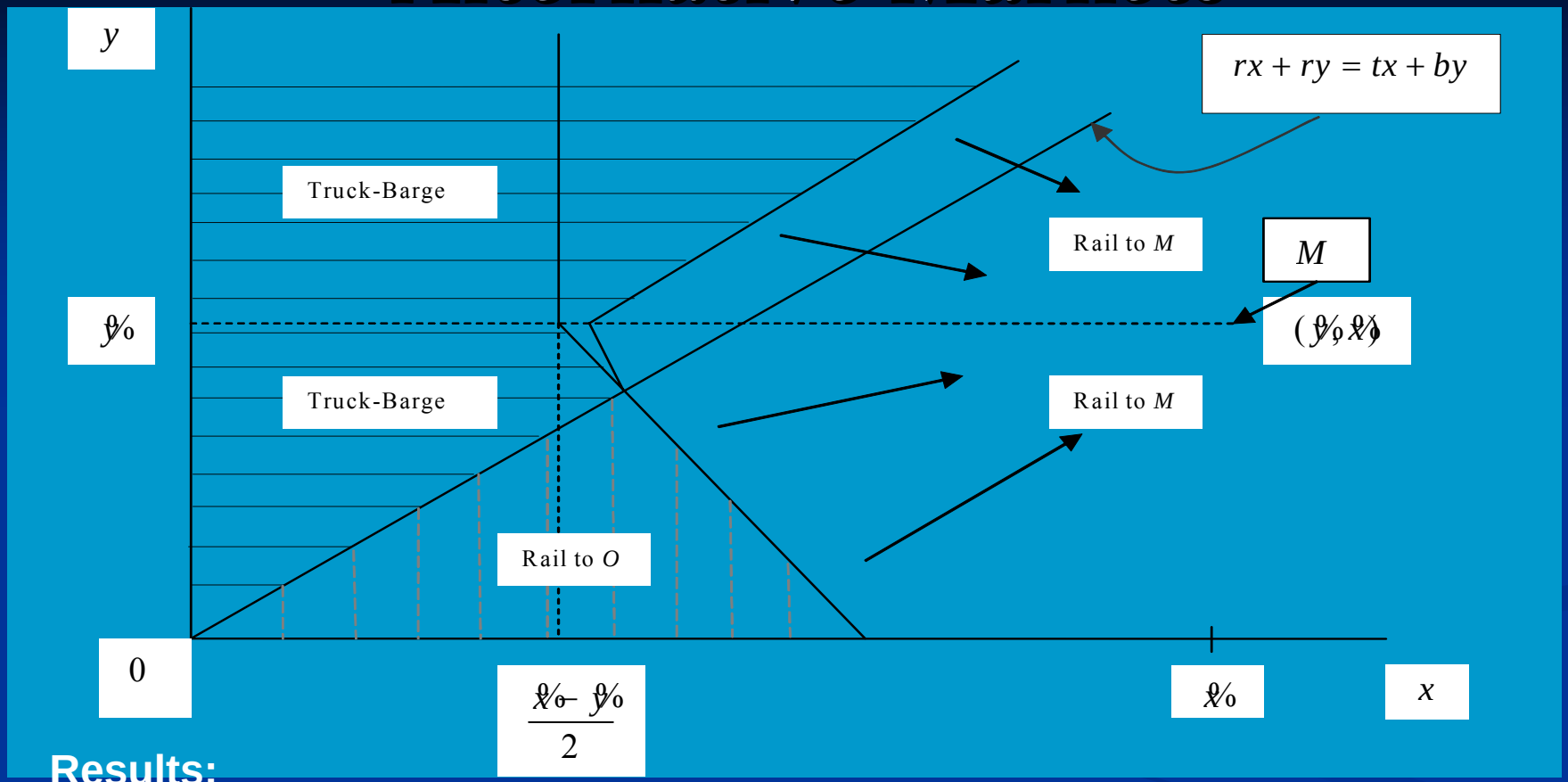
Farmers bind rail pricing with a reservation price



Results

1. Until farmers' reservation prices are hit, same results as before.
2. When reservation price is binding, rail markups fall until zero. At this point, they do not provide service.

Alternative Markets



Results:

1. Three options delineate the market areas
   - T-B to 0 versus Rail to 0
   - R to 0 versus R to M
   - T-B to 0 versus Rail to M
2. Market areas are determined on cost-basis and are min cost so they are efficient.
3. Rail market power is a simple transfer from shippers to rail.
4. Improvements to barge, increase barge areas and reduce rail rates.

Downward Sloping Demands

- Shippers max profit according to

$$\pi = \{p_T - p\}Q - C(Q)$$

this gives:

$$Q^* = C'^{-1}(p_T - p) = D(p)$$

The supply to the terminal market and the demand for transportation.

- Railroads max profit according to:

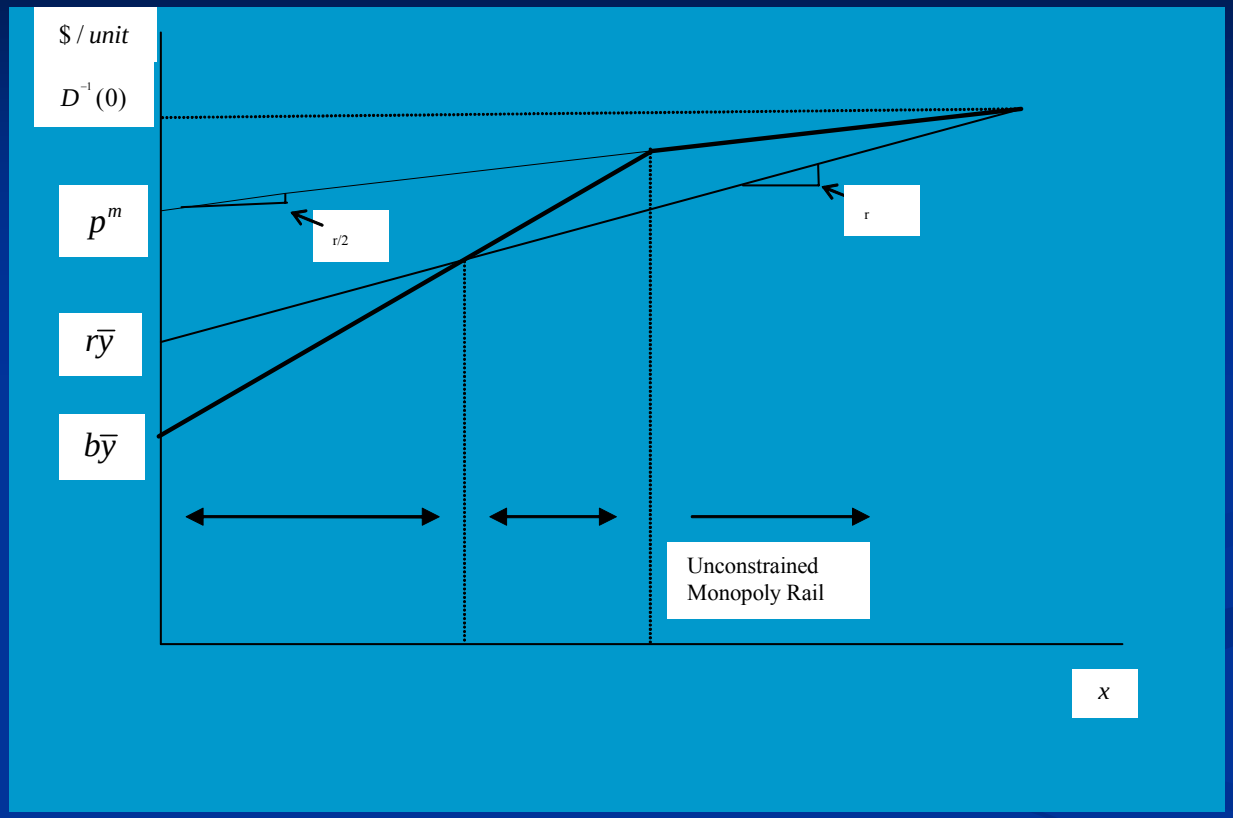
$$\pi = (p - c)D(P)$$

- with first order conditions

$$(p - c)D'(p) + D(p) = 0$$ if they are unconstrained.

$$p = tx + by$$ If constrained by truck-barge

Downward Sloping Demands and RR Pricing



Results (improvement in barge):

1. Direct benefit of cost reduction for barge users and an increase in the natural barge market.
2. Reduction in rail rate two components:
   a. A transfer from rail to farmers
   b. A reduction in DWL

Rail Capacity Constraints

- The basic starting point is that the constraint is such that the RR cannot serve all of its natural market and so must allocate capacity across that region.
- The constraint is on the number of miles that can be traveled over some time period.
- The railroad will allocate the capacity to the points that yield the highest markup.
- Let the markup per-mile be:

$$m(y,x)=\frac{(tx+by)-(rx+ry)}{(x+y)}$$

$$\mathrm{sgn}\left(\frac{dm}{dx}\right)=(t-b)y>0$$ Markup increases with distance from the river

$$\mathrm{sgn}\left(\frac{dm}{dy}\right)=(b-t)x<0$$ Markup falls with distance from the terminal market

- To bring these together, fix m, and solve for *y*

$$y = \frac{t - r - k}{-b + r + k} x$$

- These are the locus of *y's* and *x's* that yield a given markup. Now, reduce the markup until the capacity constraint is met. This gives the capacity constrained rail market area which is smaller than the unconstrained area.

- If barge rates increase and the capacity constraint is binding, the effect will be to increase the railroads markups. It will not change the market area.

Related Work

- Equilibrium model of barge market with spatial competition from rail and congestion in the lock system.
- Welfare analysis in spatial models. A comparison of measures from ACE planning models, S-TJ models and a "full-spatial" model.
- The introduction of infrastructure and and strategic behavior between rail, barge and shippers over space (Bertrand and Cournot).